[9] "Synthetic sapphire, an optical material," Linde Industrial Crystals Bull. F-917B (October 2, 1961).

[10] Nerem, R. M., "Stagnation point heat transfer in high enthalpy gas flows. Part II: Shock layer radiative emission during hypervelocity re-entry," Research and Technology Div., Air Force Systems Command FDL-TDR-64-41 (May 1964).

[11] Nerem, R. M. and Stickford, G. H., "Radiative and convective heating during hypervelocity re-entry," AIAA J. 2, 1156–1158 (1964).

An Investigation of Cylindrical Starting Flows

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When a large pressure discontinuity is suddenly applied across a two-dimensional orifice or slit, a nonsteady flow is created which later develops into a steady, rapid expansion. An experimental program was undertaken to study this expansion and evaluate it as a possible means of generating electronic population inversions.[1] A by-product of this study was a brief investigation of the early stages of the flow, when it is dominated by a cylindrical blast wave. The latter work is reported here. The results were presented in abbreviated form in Ref. 2.

The orifice, which was 2 mm high and approximately 3 in. wide, was cut in the end wall of a shock tube, so that the configuration was essentially that of a shock tunnel with a nozzle of semiangle $\simeq 90°$. Qualitative studies of the flow were made using schlieren photography. An example is shown in Fig. 1. The gas (air) is shown flowing through the orifice from left to right, and the flow is bounded by a strong cylindrical shock traveling outward into the low-pressure region. Other flow details that are indicated are an inner shock wave typical of an overexpanded nozzle flow and the two separated flow regions. The latter feature a mixing zone and an oblique shock wave caused by the separation. The interface separating gas initially upstream of the orifice from that downstream

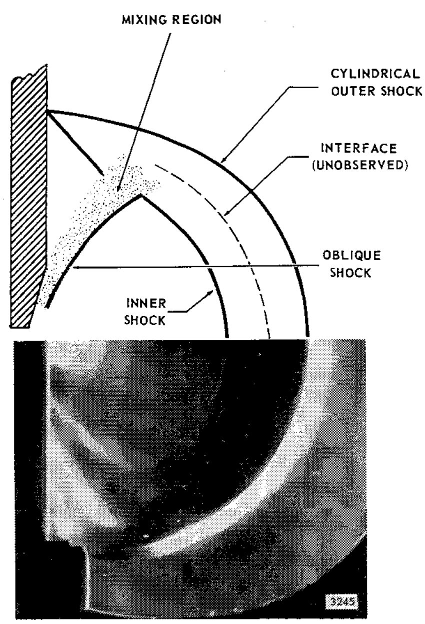

Fig. 1 Orifice starting flow.

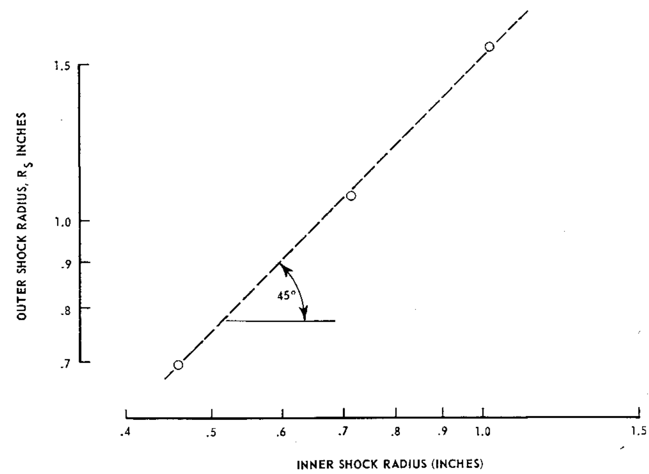

Fig. 2 Flow scaling.

is indicated, although it was not observed experimentally. The edge of the orifice is shown to be beveled, and, although the expansion angle is very large (~80°), the flow tended to follow the surface. Photographs taken with an unbeveled orifice plate did not show this feature; separation occurred at the orifice edge. It should be stressed that these observations were for flows for which the separation angle was less than 80°. For the photomultiplier studies described below, the unbeveled configuration was always used.

The schlieren photographs revealed an interesting and important feature of the flow. That is, for fixed conditions, photographs taken at different times were scaled replicas of each other. This is demonstrated in Fig. 2, which shows the radius of the outer shock plotted against that of the inner shock, both being measured along the centerline. Over the range indicated (a doubling of the shock radii), the proportionality is very good. On the basis of this evidence, one might expect that the major portion of the flow field is self-similar. Thus, explicit time dependence can be eliminated from the description of the flow by referring all radial distances to the outer shock radius. This point is discussed later. Photographs taken under different conditions did not scale; this is also considered later.

Complementing the study described previously was a quantitative investigation of the growth of the outer shock. At early times this shock is strong, and immediately behind it the processed gas is radiating, permitting the use of photomultipliers as time-of-arrival gages. One of these, receiving collimated light from the edge of the orifice, recorded the emergence of the flow. A second one, receiving light from a known distance downstream, recorded the arrival there. The outputs were displayed on a dual-beam oscilloscope, a record being shown in Fig. 3. The signal preceding the arrival at the downstream station is due to scattered light.

Before any results are presented, consider now a mathematical description of the situation based on two-dimensional

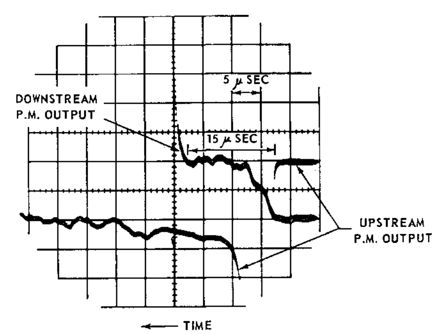

Fig. 3 Typical oscilloscope record.

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blast-wave theory[3] with a single modification. The essential details are as follows.

The total energy of the flow field, that is, the energy of the region bounded by the outer shock on one side and the orifice plate on the other, may be written

$$E = \int_{-(\pi/2)}^{\pi/2} \int_0^{R_s} \left[e + \frac{1}{2}(u^2 + v^2) \right] \rho r dr d\theta \qquad (1)$$

where e is the specific internal energy, u and v are the velocity components, and ρ is the density.

The similarity assumption is now introduced by the following substitutions:

$$\eta = \frac{r}{R_s} \qquad u = \dot{R}_s \bar{u} \qquad v = \dot{R}_s \bar{v}$$
$$e = \dot{R}_s^2 \bar{e} \qquad \rho = \rho_0 \bar{\rho}$$

Then,

$$E = R_s^2 \dot{R}_s^2 \rho_0 \int_{-(\pi/2)}^{\pi/2} \int_0^1 \left[\bar{e} + \frac{1}{2}(\bar{u}^2 + \bar{v}^2) \right] \bar{\rho} \eta d\eta d\theta \qquad (2)$$

It can be shown that explicit time dependance of the equations of motion can be eliminated if

$$R_s \sim t^N \qquad (3)$$

where t is time. Also, since the orifice is choked,

$$E = \zeta h t \qquad (4)$$

where ζ is the energy flux through the orifice and h is the orifice height. Equation (4) is the point of departure from the usual blast-wave theory in which E is supposed constant. The present approach is analogous to the steady hypersonic flow over a slender body whose drag is proportional to its length.[3]

Equations (2–4) lead to

$$R_s \sim (\zeta h/\rho_0)^{1/4} t^{3/4} \qquad (5)$$

It is implicit in the derivation that the integrand of Eq. (2) is a universal constant. However, the previous discussion of the schlieren photographs leads to the conclusion that, although the integrand is independent of time, it does depend on the operating conditions. Thus, only the time dependence predicted by Eq. (5) can be expected to be correct. The value of N for the constant energy case is $\frac{1}{2}$; if total momentum flux, rather than energy flux, is conserved, an answer of $\frac{2}{3}$ is obtained.

In the light of the previous predictions, the experimental results can now be considered. Figure 4 shows the shock radius in inches plotted against time in microseconds for two sets of conditions. The initial pressure in the shock tube is P_1, and therefore, since there was no orifice diaphragm, it is the pressure into which the gas from the orifice expands. The Mach number of the driving shock wave is M_1. For each set of points, two lines have been drawn matching the data at arbitrary points. The solid line corresponds to energy conservation and therefore has a slope of $\frac{3}{4}$, whereas the broken line corresponds to momentum conservation and has a slope of $\frac{2}{3}$. Clearly, the experimental points correspond closely to these values of N. The scatter and experimental uncertainty preclude the favoring of either of these values, but it should be pointed out that the conservation of energy in blast-wave problems is favored by both experiment and more refined theories.[4, 5] Two other lines are shown on Fig. 4 for comparative purposes. One represents the constant energy solution, and the other represents the acoustic limit.

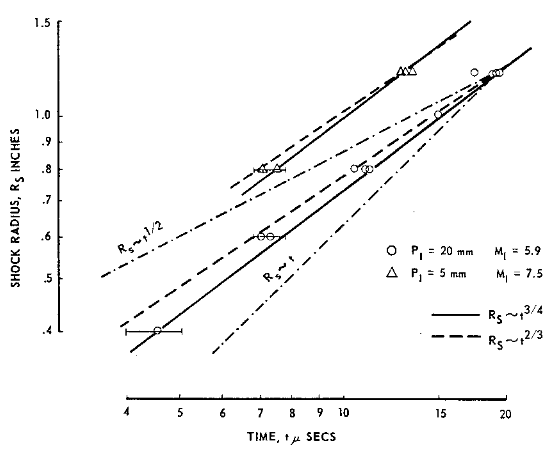

Fig. 4 Shock radius vs time history.

The calculated scaling factor for the two sets of data is approximately 1.1 in the shock radius and is clearly far too small. This lack of agreement is consistent with the earlier observations.

The evidence presented so far strongly favors the existence of flow-field similarity. However, within the nonsteady flow, a steady flow region must develop. At very early times, this will be bounded by a Mach wave of velocity $(u - a)$. After a few microseconds (much less than the 10–20 μsec flow time associated with Fig. 2), this wave will cross the inner shock, and this shock wave will then become the boundary between the steady and nonsteady flow. Now a steady flow cannot be matched with a self-similar flow across this shock wave. Consequently, a satisfactory explanation of the observed scaling is yet to be forthcoming, and further experimental work seems desirable. It should be noted that this point does not throw doubt on the blast-wave analysis, since the major portion of the energy resides close to the outer shock, between the shock and the interface.

References

[1] Hurle, I. R., Hertzberg, A., and Buckmaster, J. D., "The possible production of propulsion inversions by gasdynamic methods," Cornell Aeronautical Lab. Rept. RH-1670-A-1 (December 1962).

[2] Buckmaster, J. D., "The structure of flow produced by a sudden adiabatic expansion through a sharp-edged slit," American Physical Society Divisional Meeting, Paper I2. (November 1963).

[3] Lin, S. C., "Cylindrical shock waves produced by instantaneous energy release," J. Appl. Phys. **25**, 54–57 (1954).

[4] Rae, W. J. and Kirchner, H. P., "A blast-wave theory of crater formation in semi-infinite targets," *Proceedings of the 6th Symposium on Hypervelocity Impact* (Firestone Tire and Rubber Co., Akron, Ohio, August 1963), Vol. 2, Part 1, pp. 163–227.

[5] Taylor, G. I., "The formation of a blast wave by a very intense explosion, II. The atomic explosion of 1945," Proc. Roy. Soc. **A201**, 175–186 (1950).

Equilbrium Turbulent Boundary Layers

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FOR the past few years, the writer and D. M. Gibson have been engaged in the study of equilibrium turbulent boundary layers. An early version of the work appeared in 1962[1] but the final work is only recently available.[2, 3] Since the work has been so long in evolving and since it may not be available in the open literature for a while yet, a short summary will be provided here. This summary has been prompted particularly by the recent work of Libby, Baronti, and Napolitano[4] where, indeed, reasonable predictions of data have been accomplished. However, in comparison, the writer believes his own work to be analytically more precise, physically simpler, and amendable to further extensions. Moreover, unlike Refs. 1 and 4, Refs. 2 and 3 include cases of

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